# V2X HUB Average Delay Logic

This document outlines the estimation of average traffic delay for V2X Hub. In traffic engineering, three distinct components of delay are defined that include various traffic conditions at signalized intersections. These components are uniform delay, random delay, and overflow delay. Different analytical models are proposed for each of these components that typically take macroscopic traffic measures (e.g., ratio) and signal timing parameters as inputs. The average traffic delay is then determined as a linear combination of values of the delay components. In the connected environment, the average delay can be directly obtained with the vehicle trajectories. This dynamic microscopic approach is capable of estimating average traffic delay in real-time that includes all mentioned components.

## Requirements and Assumptions

* Connected vehicles are equipped with OBUs.
* Connected vehicles should be able to frequently send BSMs to V2X Hub through an RSU when entered to a certain vicinity of the RSU.
* V2X Hub should be able to frequently receive SPaT messages from the signal controller.

## Inputs to the Average Delay Estimation Algorithm

* BSM
  + Vehicle location
  + Vehicle speed
  + Vehicle acceleration
  + Vehicle size
* MAP
  + Roadway geometry
* SPaT
  + Real-time signal statues (i.e., red or green)
* Parameters
  + RSU detection space range
  + Stopping speed range (~1 m/s)
  + Default backward shock-wave speed 1 (depends on traffic demand, see Figure 2)
  + Default backward shock-wave speed 2 (~ 10.5 m/s, see Figure 2)
  + Safe gap distance (~ 1.5 m)
  + Time interval

## Steps to the Average Delay Estimation

As the inputs are continuously fed into V2X Hub, the average delay estimation algorithm below is repeated at every predetermined time interval. Also, these steps are applied to each intersection direction separately. Let and define the current time and time interval duration, respectively. Further, let define the number of lanes.

1. Update the list of connected vehicles located in the RSU range for each lane.
2. Update vehicle positions, speeds, accelerations, and sizes in the lists. BSM messages include vehicle positions formatted as GPS coordinates (i.e., latitudes and longitudes). These coordinates should be converted into a longitudinal coordinate system that decreases as the vehicle approach the intersection and the origin of the coordinate system is set to the stop bar location. Each direction can be assigned to one longitudinal coordinate system. Figure 1 illustrates the longitudinal coordinate system along a direction of a roadway.

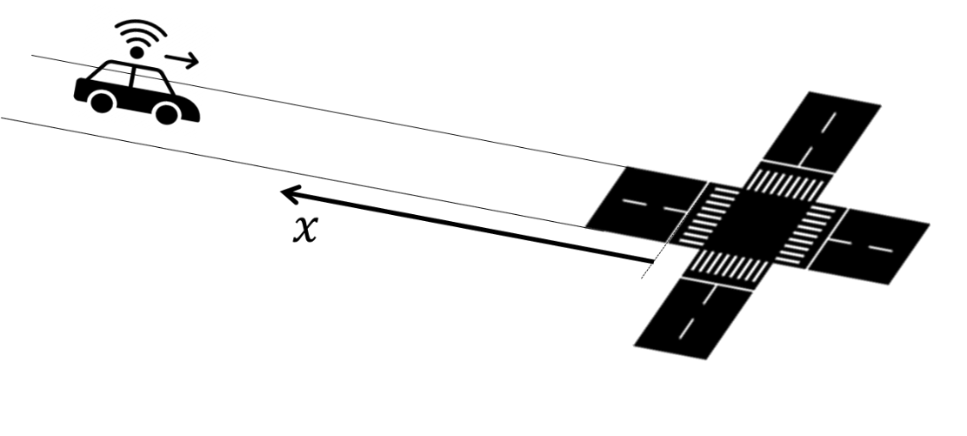


Figure 1 Longitudinal coordinate system (source: FHWA).

1. Sort vehicles according to their distance from the stop bar or their values in the longitudinal coordinate system in ascending order for each lane. For system efficiency, V2X Hub may save the order obtained from the previous algorithm iterations, and update that as a vehicle enters the range, change lane, or departs from the intersection.
2. Update vehicle states and the corresponding state change times for each lane. For each vehicle in the lists, we define two states: moving and stopping. A vehicle is set to the stopping state when its speed drops to a value less than the stopping speed range parameter, and moving state otherwise. Depending on the quality of acceleration profiles provided by the BSM messages, we may be able to use both speed and acceleration characteristics to classify vehicle states. V2X Hub records the vehicle states and the corresponding state change time, defined as stopping and accelerating times and adds them to the lists. The accelerating times are only defined when the signal is green and thus the vehicles start accelerating to traverse through the intersection.
3. Update the shockwave speeds. We define two backward shockwaves. The first one separates the change of vehicle states from the moving to stopping states as vehicles approach the intersection. The second one separates the state changes from stopping to moving as vehicles start accelerating when the signal turns green. Figure 2 illustrates the two backward shockwaves in a space-time diagram. This figure shows vehicle trajectories for a stream of vehicles approaching a signalized intersection. The traffic signal status is shown as red, green and yellow bars at the intersection (shown at top of the figure). A portion of vehicles in this figure are connected (OBU-equipped) vehicles, which their trajectories are shown as solid red curves, and the remaining vehicles with dashed-blue trajectories are not connected. The non-connected vehicle trajectories are invisible to the RSU. However, the location of connected vehicles can be broadcasted to the RSU at any time once they are in the RSU detection range. The 4-point orange and 5-point green stars in Figure 2 indicate the stopping and accelerating space-time points, respectively. These points actually separate the vehicle states in the space-time diagram. Connecting the stopping space-time points together yields shockwave 1, as shown as dashed red line. Similarly, we can obtain shockwave 2 by connecting the accelerating space-time points together, shown as dashed green line.

The speeds of these shockwaves, which are the absolute values of the shockwave slopes in the space-time diagram can be determined by fitting a linear regression model to the corresponding points in the space-time diagram, calculated as

where is the shockwave speed at lane , is the number of stopping/accelerating points used for estimating the speed of shockwave speed at lane , and and are the time and location of point along shockwave speed at lane , , respectively. Note that in order to estimate the shockwave speeds with the linear regression formula, at least two stopping/accelerating points are required for each of the shockwaves. In case that only one stopping/accelerating point is available for each of the shockwaves, the corresponding shockwave speed(s) is(are) set to the default value(s) defined in the set of parameters. For shockwave 1, different default values can be estimated using offline historical traffic data for different times of a day during weekday, weekend, and special event periods or can be estimated in real-time using the estimated traffic flow. However, shockwave 2 default value can be set to a single static value that depends on the intersection saturation flow rate and jam density, which can be also calibrated using historical traffic data.

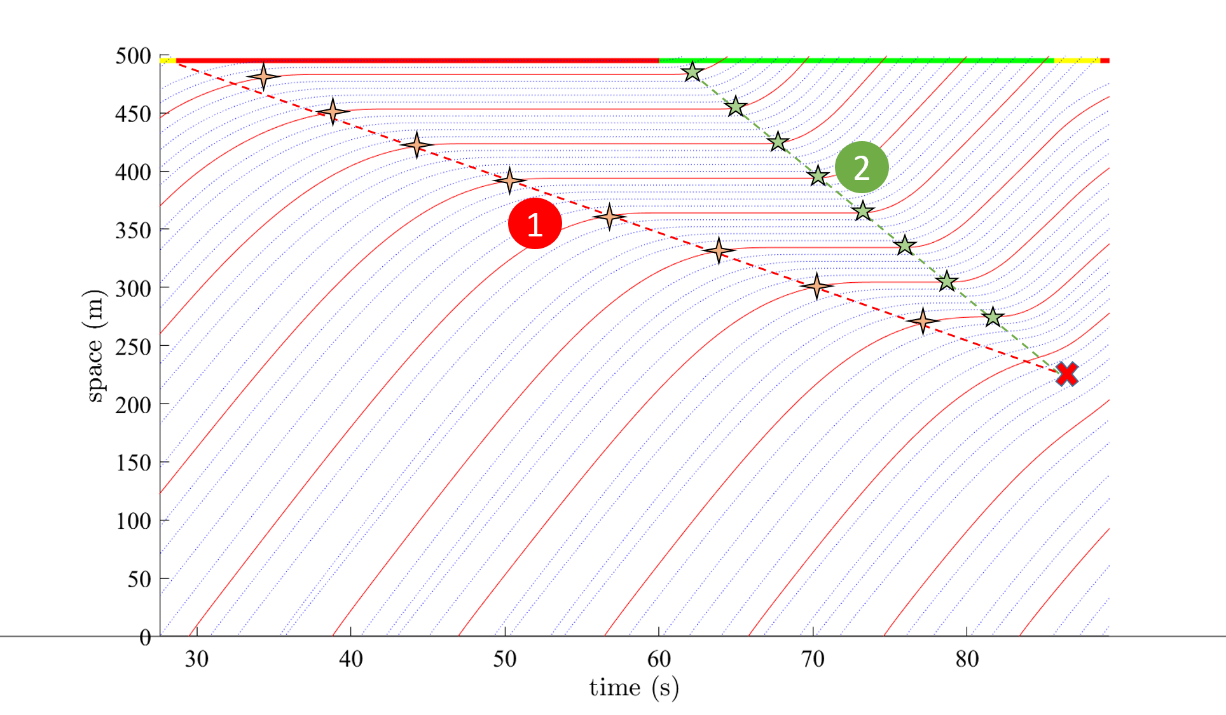
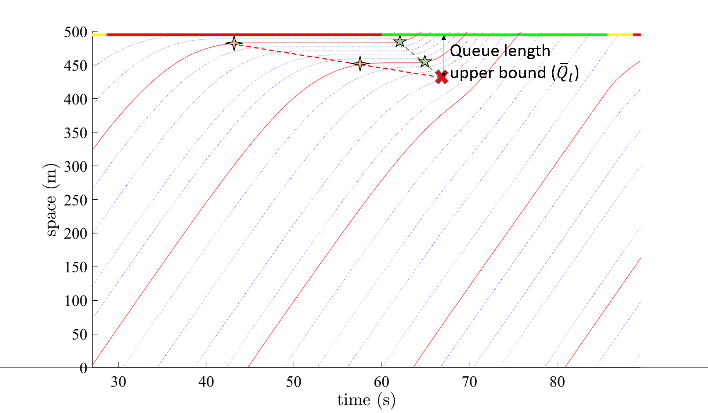
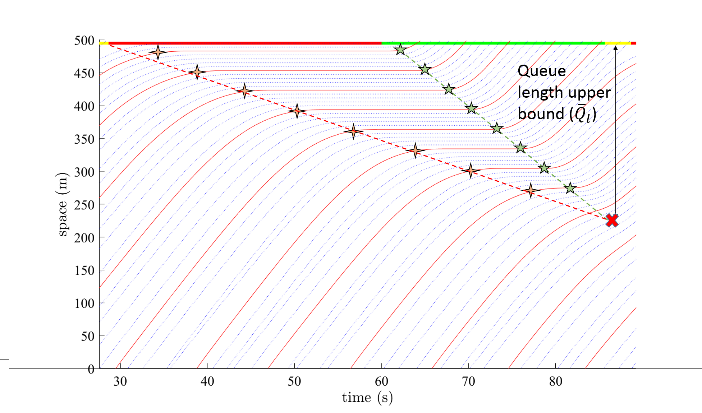


Figure 2 An illustration to backward shockwaves (source: FHWA).

1. If the signal status is green, then update the queue length upper bound. It can be shown that the queue cannot spill back farer than the intersection between the two shockwaves illustrated as red multiply symbol in Figure 2. This upper bound can be updated once the signal turns into green, and shockwave speeds 1 and 2 are estimated. Let denote the queue length upper bound at lane that is calculated as follows for all . Figure 3 illustrates estimation for two traffic scenarios: relatively high and low demand cases, as shown in Figure 3(a) and Figure 3(b).

where is the shockwave function intercepts at lane for , and is the signal red time interval that applies to lane (e.g., through or left movement red signal).



(a) (b)

Figure 3 An illustration to queue length upper bounds in a) high demand and b) low demand scenarios (source: FHWA).

1. Estimate the queue length. The corresponding distance to the stop bar of the last stopped vehicle provides a lower bound to the overall queue length. However, it is likely that there are other queued vehicles behind the last connected vehicle. Therefore, we need to extend the last known stopping location using the speed of backward shockwave 1 to obtain the queue length. The extension distance is determined by multiplying the shockwave speed 1 by the time difference between current time *t* and the last stopping time. If the signal is green, and thus a queue upper bound is available, then the estimated queue length should be compared with the upper bound and modified accordingly, otherwise we set to infinity for any . Let denote the queue length at lane , *,* which is calculated as follows. Figure 4 illustrates the queue length estimation.

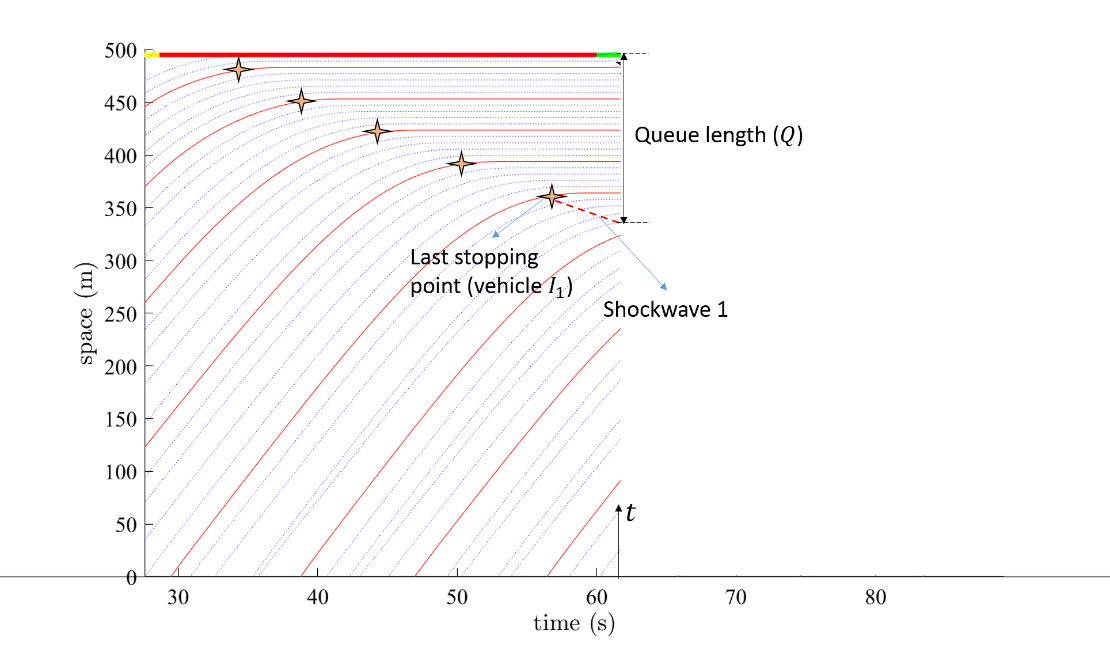


Figure 4 An illustration to queue length estimation (source: FHWA).

1. Estimate average jam spacing, denoted by . Jam spacing is defined as the bumper-to-bumper distance between a pair of consecutive vehicles stopped in the queue. Jam spacing consists of the vehicle length and the safe gap distance between vehicles. While the safe gap distance can be considered as a constant value (~ 1.5 m), vehicle lengths may vary among different vehicles. Therefore, the average jam spacing can be determined by adding the constant safety gap distance to the average vehicle size across all lanes.
2. Calculate the total delay area. The total delay area in this document is defined as the area bounded by the vertical lines at times and , the horizontal line at the stop bar location, and shockwave 1 and 2 in the space-time diagram. This area is illustrated as the hashed-red zone in Figure 5 for different time intervals. Note that the intersection clearance and start-up lost times are inherently considered in the shockwaves function intercepts. Let denote the total delay area for lane .

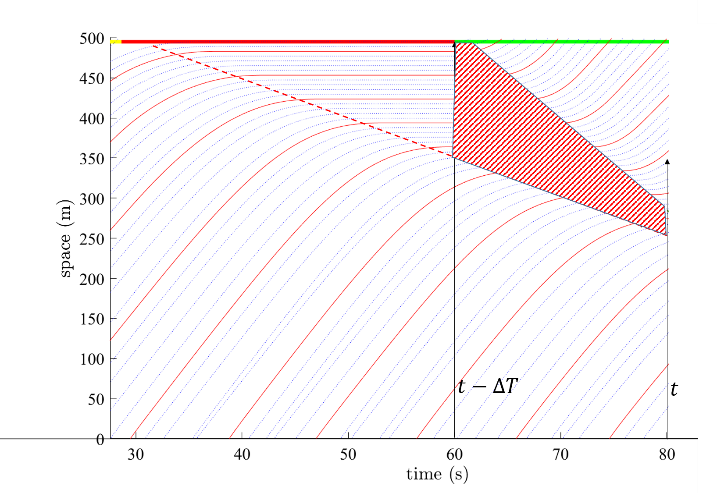
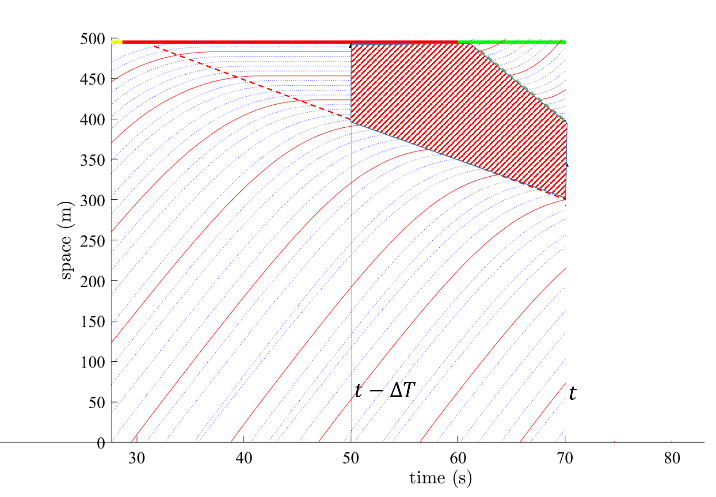
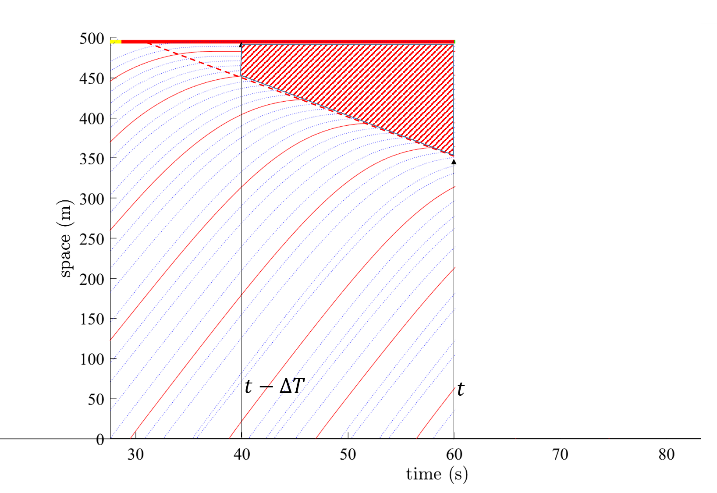
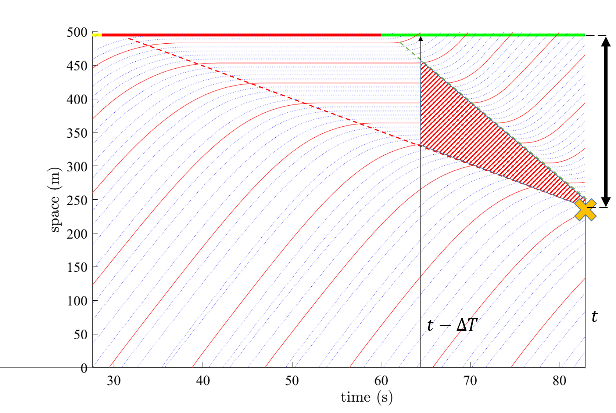
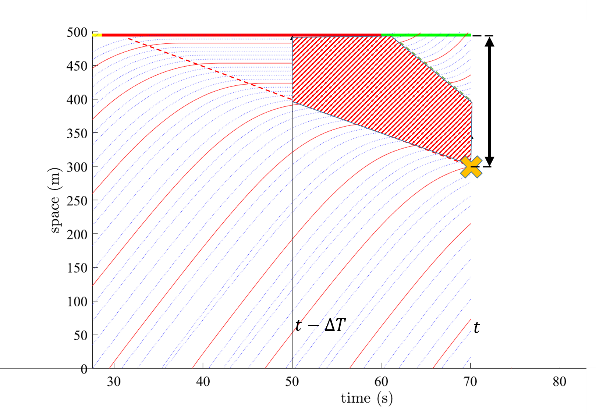


Figure 5 An illustration to total delay area calculation (source: FHWA).

1. Estimate the total delay for each lane , denoted by . The total delay for each lane and at a certain time interval can be approximated by dividing the delay area to the average jam spacing:
2. Estimate the number of delayed vehicles for each lane , denoted by . The total number of delayed vehicles during time interval can be estimated by dividing the queue length at time (as illustrated in Figure 6) by the average jam spacing:



(a) (b)

Figure 6 An illustration to distance between back of the queue at time t and the stop bar (source: FHWA).

1. Estimate the average delay, denoted by . The average delay at any time interval is defined by dividing the total delay at all lanes by the total number of delayed vehicles:

Note that if , then can be set 0.